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Optimal Interpolation and Isarithmic Mapping of Groundwater Salinity in Tebrak Area, Central Saudi Arabia

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Abstract. The principle of optimal interpolation using regionalized variable theory is applied to the spatial analysis of groundwater salinity. Interpolated values of groundwater salinity for Tebrak area in central Saudi Arabia were obtained by the kriging technique in a GIS compatible format and isarithmic map of the kriged values of groundwater salinity was drawn to depict the regional trend of groundwater salinity in the area. The map shows an elongated region of low groundwater salinity extending through the center of the sampled area from northwest to southeast. The estimation error, depicted in the isarithmic map of kriging variance is small in the interior region, where sufficient data were available, which implies high confidence on the interpolated values of groundwater salinity, and increases at the margins.

Introduction

Groundwater salinity is an important factor affecting the economical feasibility of irrigated agriculture in arid and semi-arid regions. Groundwater salinity maps when integrated with soil salinity maps in the Geographic Information Systems (GIS) lead to optimal allocation of irrigation projects. This requires computer generated groundwater salinity data that can be readily integrated in GIS analysis. Since obtaining groundwater samples in a fine regular grid is impossible, an accurate interpolation technique to define groundwater salinity at each node of a regular grid from spatially scattered observations is needed.

Variations in groundwater salinity tend to be spatially correlated. That is, two values taken close together tend to be more alike than two observations far apart. The theory of regionalized variables, developed for mining and mineral exploration, makes use of the spatial interdependence in the limited data available to estimate the variable of interest at places where measurements have not been made due to cost of determination and/or initial degree of interest. The estimation proceeds in two main stages. The first is a spatial analysis of the limited data with the results expressed in the form of variograms. The second is interpolation using the experimental variogram by a procedure known as kriging. Local estimates obtained by the kriging procedure are optimal in the sense that they are unbiased and have a minimum variance.(1) Unlike most spatial interpolation techniques used in natural resource surveys, kriging quantifies the spatial dependency by a variogram and provides estimates of the estimation variance which indicate the reliability of the results. Recent works(2) indicate that kriging outperforms other techniques when reasonable data points are available.

The objective of this paper is to demonstrate the usefulness of the kriging technique for groundwater salinity analysis as well as to produce an isarithmic map of groundwater salinity for Tebrak area to depict the trends in groundwater salinity.

Method

The variogram γ(h) for the regionalized variable Z is defined as

$$\gamma(h) = \frac{1}{2} var\left[Z(x) - Z(x+h)\right] \tag{1}$$

where Z(x) is the value of Z at location x and Z(x+h) is the value of Z at location x+h and var[] is the variance operator. An estimate of γ is γ^* given by

$$\gamma^*(h) = \frac{1}{2n(h)} \sum_{i=1}^{n(h)} \left[Z(x_i) - Z(x_i + h)\right]^2 \tag{2}$$

where n(h) is the number of pairs separated by distance h. Kriging calculates the estimated value of the unknown variable Z^* at some point x_0 using a weighted average of the known values of the variable Z at points x_i:

$$Z^*(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i) \tag{3}$$

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1) A. McBratney and R. Webster, "Optimal Estimation and Isarithmic Mapping of Soil Properties V. Co-Regionalization and Multiple Sampling Strategy." *J. Soil Sci.*, 34(1983), 137-162; A. Warrick, D. Myers and D. Nielsen, "Geostatistical Methods Applied to Soil Science," in A. Klute, ed. *Methods of Soil Analysis, Part I. Physical and Mineralogical Methods*, agronomy monograph no. 9, 2nd ed. (Madison, Wis. Soil Science Society of America, 1989).

2) L. Hughson, D. Huntley and M. Razack, "Cokriking Limited Transmissivity Data Using Widely Sampled Specific Capacity from Pump Tests in an Alluvial Aquifer," *Ground Water*, 34, No. 1(1996),12-18; E. Hosseini, J. Gallicand and D. Marcotte, "Theoretical and Experimental Performance of Spatial Interpolation Methods for Soil Salinity Analysis," *Trans. ASAE*, 36, No. 6(1994),1799-1807; T. Burgess. and R. Webster, "Optimal Estimation and Isarithmic Mapping of Soil Properties. I. The Semi-Variogram and Punctual Kriging," *J. Soil Sci*. 31(1980), 315-31

where n is the number of measured Z values, and λ_i is the weighing factor applied to the known value of Z at point x_i. The optimal estimate of the unknown true value of Z at x_0 ($Z(x_0)$) is found by choosing the weight factors λ_i such that

$$E\left[Z^*(x_0) - Z(x_0)\right] = 0 \tag{4}$$

$$var\left[Z^*(x_0) - Z(x_0)\right] = a \ \ minimum \tag{5}$$

where E[] is the expectation operator. The condition that guarantees Eq.(4) to hold is

$$\sum_{i=1}^{n} \lambda_i = 1 \tag{6}$$

The weight factors λ_i that satisfy both Eqs.(4) and (5) are given by

$$A\begin{bmatrix}\lambda \\ \mu\end{bmatrix} = b \tag{7}$$

where A is an n+1 by n+1 matrix:

$$A = \begin{bmatrix} \gamma^*_{1,1} & \gamma^*_{2,1} & . & . & . & \gamma^*_{n,1} & 1 \\ \gamma^*_{1,2} & \gamma^*_{2,2} & . & . & . & \gamma^*_{n,2} & 1 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \gamma^*_{1,n} & \gamma^*_{2,n} & . & . & . & \gamma^*_{n,n} & 1 \\ 1 & 1 & . & . & . & 1 & 0 \end{bmatrix} \tag{8}$$

where $\gamma^*_{2,1}$ is the variogram evaluated using the experimental variogram model for the distance between observations 2 and 1. The weights column vector is

$$\begin{bmatrix}\lambda \\ \mu\end{bmatrix}^T = \left[\gamma_1 \gamma_2 \ldots \gamma_n \mu\right] \tag{9}$$

where μ is a Lagrangian multiplier and T stands for the transpose of. The column vector b is defined as

$$\mathrm{b}^T = \left[\gamma^*_{1,0}\,\gamma^*_{1,0}\cdots\gamma^*_{1,0}\,1\right] \tag{10}$$

where $\gamma^*_{1,0}$ is the variogram for the distance separating the location of observation 1 from the location where Z need to be estimated. The kriging variance is given by

$$\sigma^2_E = \mathrm{b}^T \begin{bmatrix} \lambda \\ \mu \end{bmatrix}. \tag{11}$$

The functional form of the variogram model needed to calculate the variogram at a given distance can be determined experimentally by plotting variograms calculated by Eq.(2) from measured data against the corresponding lag (h).

Study Area

Tebrak area is located around 24° 23' N and 45° 53' E, 100 km west of the capital city of Riyadh, in central Saudi Arabia (Fig. 1). It represents part of the Minjur formation outcrop. Soils are entisols and aridisols with variable texture ranging from sandy to loamy clay. Annual precipitation is about 100 mm and annual potential evapotranspiration exceeds 2500 mm.

The Minjur formation is mainly massively bedded, coarse to very coarse quartzitic sandstone of continental origin. Thin layers of limestone, shale, conglomerate, and gypsum are also present. The Minjur outcrop is some 300 m thick in the study area and underlain by the calcareous Jilh formation.(3) Minjur aquifer is one of the main aquifers in the country. It is utilized for both agricultural and urban uses. Utilization of Minjur aquifer in Tebrak area is primarily for agriculture where wheat is the main crop.

Data Description

The data set consists of 50 observations of measured groundwater electrical conductivity. Groundwater samples were obtained from abstraction wells that are screened only in the Minjur aquifer. Samples were collected during the first week of September 1997 directly from freshly tapped water after several hours of pumping. Well location is determined by Global Positioning System (GPS) and groundwater electrical conductivity is measured with a high-accuracy temperature-compensating conductivity

3) Ministry of Agriculture and Water, *Water Atlas of Saudi Arabia* (Riyadh: Ministry of Agriculture and Water, 1984.)

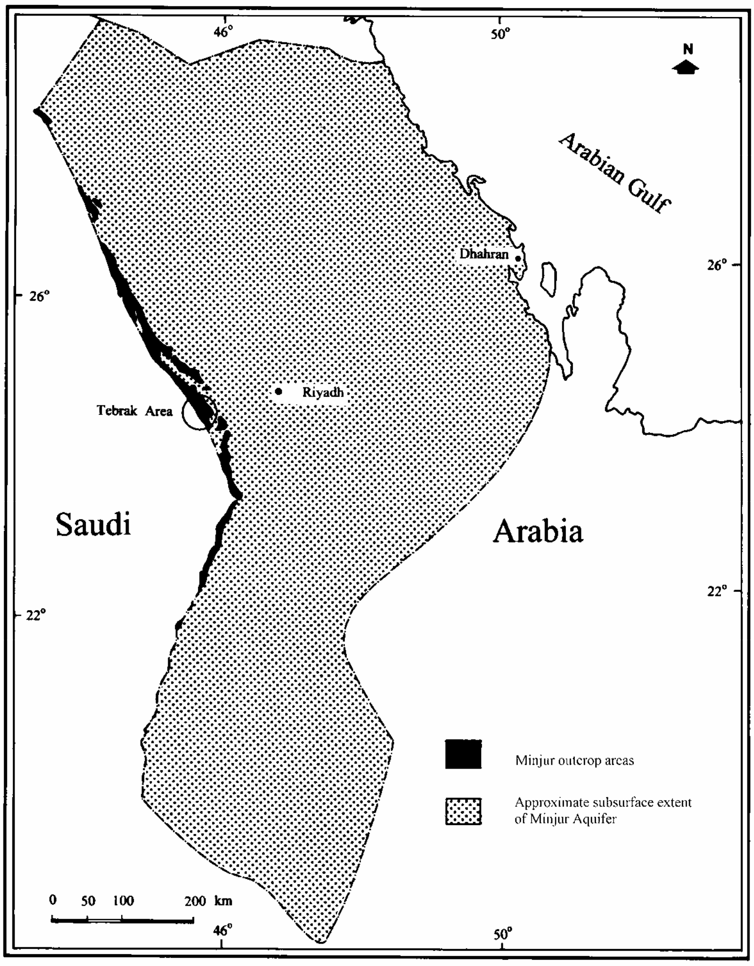

Fig. 1. Location of the study area.

meter. The spatial distribution of the sampled wells is shown in Fig. 2 and a summary statistics of measured groundwater salinity is provided in Table 1.

Table 1. Summary statistics of measured groundwater electrical conductivity (mS cm^{-1}).

Parameter	Mean	Median	Mode	Standard dev.	Skewness	Kurtosis	Range	Minimum	Maximum
Value	2.42	2.50	2.07	0.50	0.12	-0.61	2.06	1.48	3.54

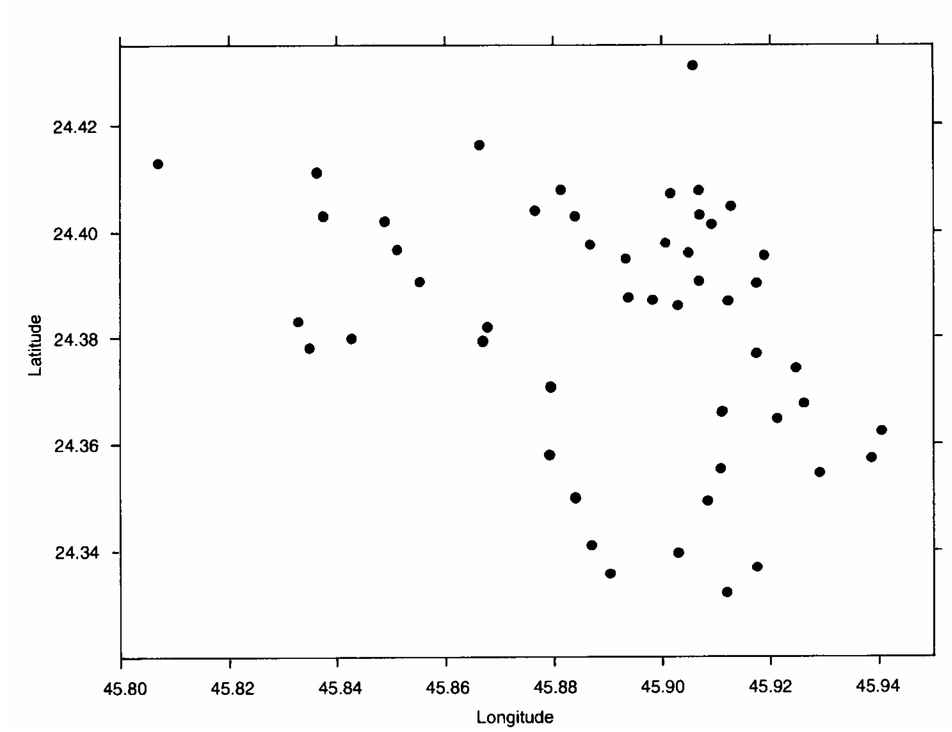

Fig. 2. Spatial distribution of sampled wells.

Results and Discussion

Variogram function

The variogram model that best fits the experimental variograms of groundwater salinity is the spherical model (Fig. 3):

$$\gamma^{*}(h) = C_0 + C\left[\frac{3}{2}\left(\frac{h}{a}\right) - \frac{1}{2}\left(\frac{h}{a}\right)^3\right] \qquad 0 < h < a \qquad (12)$$

$$= C_0 + Ch \geq a \qquad h \geq a$$

where C_0 is the nugget = 0.11 (mS $cm^{-1})^2$, C_0+C is the sill = 0.27 (mS $cm^{-1})^2$, and a is the range = 4 km.

Point estimation

Kriging was performed for a fine grid and the kriged values of groundwater salinity were generated in a raster format that is GIS compatible. An isarithmic map of the kriged values of groundwater salinity is shown in Fig. 4. The map reveals an elongated region of low groundwater salinity extending through the center of the sampled area from northwest to southeast. The southwestern region of the map is characterized by sparse contour lines because the kriging procedure assign the average value (λ_i =1/n) for points separated by a distance equal to or greater than the range (a) from all the sampled points.

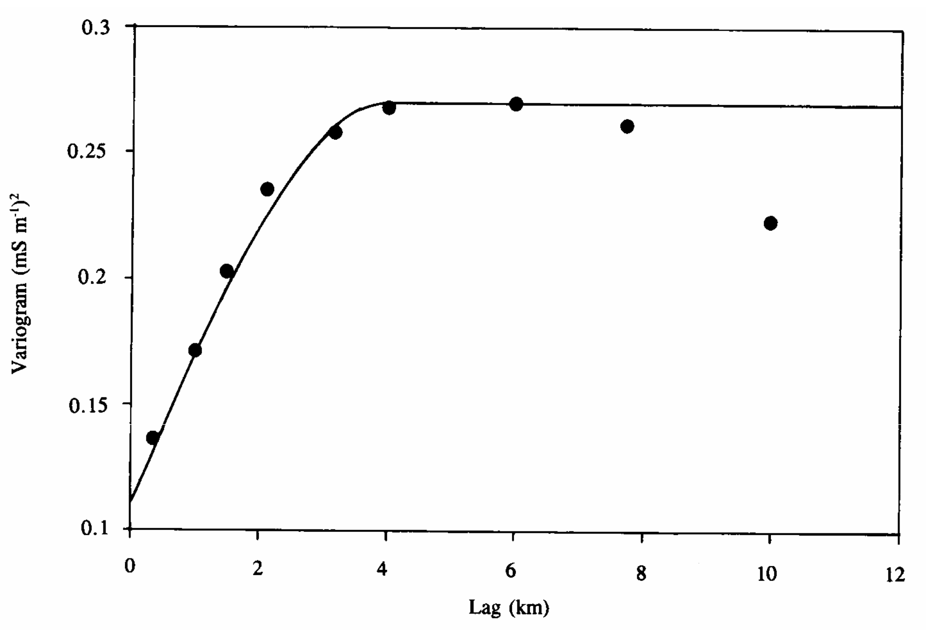

Fig. 3. Plot of experimental variograms (mS cm^{1})^{2} vs lag (km).

The spatial distribution of the kriging variance σ^2_E is mapped in Fig. 5. Kriging variance which is a function of sampling intensity and the variogram model indicates which regions having interpolated values of groundwater salinity with the greatest confidence. Its value ranged from 0.11 (the nugget) in regions very close to sampled wells to 0.27 (the sill) in places farther away. Generally, the kriging variance is small in

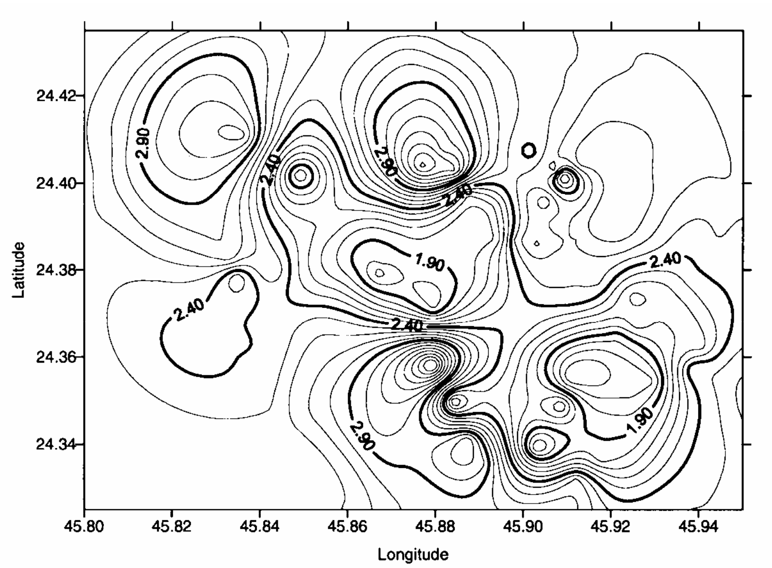

Fig. 4. Isarithmic map of groundwater salinity (mS cm[1]).

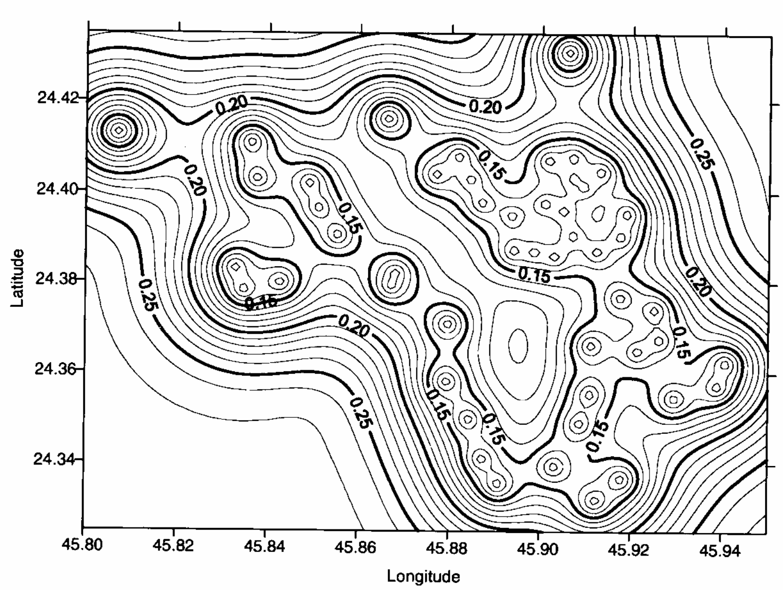

Fig. 5. Isarithmic map of kriging variance (mS cm^{-1})^2.

the interior region where sufficient samples were obtained, and increases toward the margins of the area considered.

Conclusions

Groundwater salinity is a spatially correlated phenomenon and its spatial analysis can be facilitated using the regionalized variable theory. The kriging technique provides interpolated values of groundwater salinity that are optimal and in a fine regular grid which can be incorporated in GIS analysis.

The isarithmic map of groundwater salinity interpolated by the kriging technique for Tebrak area reveals an elongated region of low groundwater salinity extending through the center of the sampled area from northwest to southeast. The estimation error, depicted in the isarithmic map of kriging variance, is small in the interior region where sufficient data were available which implies high confidence on the interpolated values of groundwater salinity.

التقدير البيني الأمثل وخريطة خطوط التساوي لملوحة المياه الجوفية في منطقة تبراك بوسط المملكة العربية السعودية

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ملخص البحث. لقط طبقت مبادىء التقدير البيني interpolation الأمثل باستخدام نظرية المتغير المؤقلم regionalized variable theory للتحليل المكاني لملوحة المياه الجوفية. تم تقدير ملوحة المياه الجوفية لمنطقة تبراك في وسط المملكة العربية السعودية باستخدام تقنية الكرجنج kriging في شكل متوافق مع أنظمة المعلومات الجغرافية GIS ، ورسمت خريطة خطوط الملوحة المتساوية لإبراز النمط الإقليمي لملوحة المياه الجوفية بالمنطقة. يظهر على الخريطة إقليم مستطيل ذو ملوحة منخفضة يمتد خلال وسط منطقة العينة باتجاه الجنوب الشرقي. خطأ التقدير – كما هو واضح في خريطة خطوط الخطأ المتساوية – صغير في المنطقة الداخلية حيث يتوافر عدد كاف من البيانات مما يدل على ثقة كبيرة في القيم المقدرة لملوحة المياه الجوفية وترتفع قيمته على الهوامش.